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$(1,2)$ S_p-Open Sets In Bitopological Spaces

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Abstract

In this paper, we introduce a new type of $(1,2)$ semi-open sets in bitopological spaces called S_p-open sets, and give some of its relationship with other types of open sets in bitopological spaces. A few properties of $(1,2)$ S_p-open sets are also studied.

Key Words:
$(1,2)$ semi-open sets, $(1,2)$ preclosed sets, $(1,2)$ S_p-open sets

INTRODUCTION

The concept of semi-open sets was introduced by Levine [5] in 1963. In 1982, Mashhour et al. [6] defined a new class of open sets which are weaker than open sets and called it as pre-open sets. Combining these two sets Shareef [8] in 2007, introduced a new class of semi-open sets called as Sp-open sets. "Bitopology" was introduced by Kelly [1] in 1963. His topology was defined by means of metrics. Later, instead of taking the topologies as by Kelly, researcher tried the study of bitopological spaces with any two topologies omitting the conditions on metrics. Almost all the properties of classical topologies were studied using the pairwise concept. Thivagar [2] in 1991, defined a new concept namely (1, 2) α-open sets. Based on (1, 2) α -open sets, Thivagar [3] in 2007, defined (1, 2)semi-open and (1, 2) pre-open sets and the main purpose of this article is to define and to generalize the (1, 2)S_P-open sets in bitopological spaces.

1. PRELIMINARIES

We recall some basic definitions and results which will be used in the next section.

Definition 1.1. [2] Let A be a subset of a bitopological space(X,τ_1,τ_2). Then A is said to be:

i. $\tau_1\tau_2$-open if $A \in \tau_1 \cup \tau_2$.
ii. $\tau_1\tau_2$-closed if $A^c \in \tau_1 \cup \tau_2$.
iii. $(1,2)\alpha$-open or ultra-open if $A \subseteq \tau_1 - Int(\tau_1\tau_2 - Cl(\tau_1 - Int(A)))$, where $\tau_1 - Int(A)$ is the interior of A with respect to the topology τ_1 and $\tau_1\tau_2 - Cl(A)$ is the intersection of all $\tau_1\tau_2$-closed sets containing A.
iv. $(1,2)\alpha Int(A)$ is the union of all $(1,2)\alpha$-open sets contained in A.
v. $(1,2)\alpha Cl(A)$ is the intersection of all $(1,2)\alpha$-closed sets containing A.

The collection of all $(1,2)\alpha$-open sets are denoted by $(1,2)\alpha O(X)$ and if this set forms a topology, then X is called an ultra space.

Definition 1.2. [4] A subset A of X is said to be:
(i) $(1,2)$ semi-open if $A \subseteq \tau_1\tau_2 - Cl(\tau_1 - Int(A))$.
(ii) $(1,2)$pre-open if $A \subseteq \tau_1 - Int(\tau_1\tau_2 - Cl(A))$.
(iii) (1,2)regular-open if $A = \tau_1 - Int(\tau_1\tau_2 - Cl(A))$.

The collection of all $(1,2)$semi-open, $(1,2)$pre-open and (1,2)regular-open sets are denoted by $(1,2)SO(X)$, $(1,2)PO(X)$ and $(1,2)RO(X)$ respectively.

Definition 1.3. [7] A subset A of a space X is said to be:
(i) $(1,2)\alpha$-closed if $\tau_1 - Cl\left(\tau_1\tau_2 - Int\left(\tau_1 - Cl(A)\right)\right) \subseteq A$.
(ii) $(1,2)$ semi-closed if $\tau_1\tau_2 - Int\left(\tau_1 - Cl(A)\right) \subseteq A$.
(iii) $(1,2)$ preclosed if $\tau_1 - Cl\left(\tau_1\tau_2 - Int(A)\right) \subseteq A$.
(iv) (1, 2) regular-closed if $A = \tau_1 - Cl\left(\tau_1\tau_2 - Int(A)\right)$.

The set of all $(1,2)\alpha$-closed, $(1,2)$semi-closed, $(1,2)$pre-closed and (1,2)regular-closed sets are defined in the usual sense and denoted as $(1,2)\alpha CL(X)$, $(1,2)SCL(X)$, $(1,2)PCL(X)$ and $(1,2)RCL(X)$ respectively. Also, for any subset A of X, the $(1,2)\alpha$-closure, $(1,2)$semi-closure, (1,2)pre-closure and $(1,2)$regular-closure of A is denoted as $(1,2)\alpha Cl(A)$, $(1,2)SCl(A)$, $(1,2)PCL(A)$ and $(1,2)RCl(A)$ respectively.

Remark 1.4. [7] For any subsets A, B of X:
(i) $\tau_1 - Int(A) \subseteq \tau_1\tau_2 - Int(A)$ and $\tau_2 - Int(A) \subseteq \tau_1\tau_2 - Int(A)$.
(ii) $\tau_1\tau_2 - Cl(A) \subseteq \tau_1 - Cl(A)$ and $\tau_1\tau_2 - Cl(A) \subseteq \tau_2 - Cl(A)$.
(iii) $\tau_1\tau_2 - Cl(A \cap B) \subseteq \tau_1\tau_2 - Cl(A) \cap \tau_1\tau_2 - Cl(B)$.
(iv) $\tau_1\tau_2 - Int(A) \cup \tau_1\tau_2 - Int(B) \subseteq \tau_1\tau_2(A \cup B)$.
(v) $(1,2)\alpha O(X) = (1,2)SO(X) \cap (1,2)PO(X)$.

Definition 1.5. [3] A bitopological space X is called an ultra T_1- space if and only if for every distinct points $x, y \in X$, there exists a $(1,2)\alpha$ −open set G containing x but not y and $(1,2)\alpha$ −open set containing y but not x.

Definition 1.6. [7] A bitopological space X is said to be $(1,2)\alpha$-hyperconnected if every $(1,2)\alpha$-open set is dense in X. That is, $(1,2)\alpha Cl(A) = X$ for any $(1,2)\alpha$ −open subset A of X.

Theorem 1.7. [7] A bitopological space X is $(1,2)\alpha$-hyperconnected if and only if $(1,2)sCl(A) = X$ for every non empty $A \in (1,2)SO(X)$.

Definition 1.8. [7] A bitopological space (X, τ_1, τ_2) is said to be $(1,2)\alpha$-extremally disconnected if every $\tau_1\tau_2$-closure of τ_1-open set is τ_1-open.

Theorem 1.9. [7] If a bitopological space X is $(1,2)\alpha$-extremally disconnected, then (1,2)$SO(X)$ forms a topology on X.

2. $(1,2)S_p$-open sets

In this section, we define the concept of S_p-open sets in bitopological spaces called $(1,2)S_p$-open sets and study some of their properties. A few properties of $(1,2)S_p$-open sets with respect to $(1,2)\alpha$-subspaces are also studied.

Definition 2.1. A $(1,2)$semi-open subset A of a bitopological space (X, τ_1, τ_2) is said to be $(1,2)S_p$-open if for each $x \in A$ there exists a $(1,2)$pre-closed set F such that $x \in F \subseteq A$.

The complement of a $(1,2)S_p$-open sets is a $(1,2)S_p$-closed set and the family of all $(1,2)S_p$-open (resp. $(1,2)S_p$-closed) subsets of X is denoted by $(1,2)S_pO(X)$ (resp. $(1,2)S_pCL(X)$).

Proposition 2.2. A subset A of a bitopological space (X,τ_1,τ_2) is $(1,2)S_p$-open if and only if A is $(1,2)$semi-open and it is the union of $(1,2)$pre-closed sets.
Proof. Obvious.

The following example shows that semi-open sets need not be S_p-open.

Example 2.3. Let $X=\{a,b,c,d\}$ with $\tau_1=\{\phi,X,\{d\},\{a,d\},\{b,d\},\{a,b,d\}\}$ and $\tau_2=\{\phi,X,\{c\}\}$. Then $(1,2)SO(X)=\{\phi,X,\{d\},\{a,d\},\{b,d\},\{a,b,d\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$ and $(1,2)S_pO(X)=\{\phi,X\}$. Here $\{d\},\{a,d\},\{b,d\}$ and $\{a,b,d\}$ are $(1,2)$semi-open sets but not $(1,2)S_p$-open sets.

The following theorem shows that arbitrary union of $(1,2)S_p$-open sets is $(1,2)S_p$-open.

Theorem 2.4. Let $\{A_\alpha:\alpha\in\Delta\}$ be a family of $(1,2)S_p$-open sets in a bitopological space (X,τ_1,τ_2). Then $\bigcup_{\alpha\in\Delta}A_\alpha$ is also a $(1,2)S_p$-open set in X.
Proof. The arbitrary union of $(1,2)$semi-open sets is $(1,2)$semi-open **[7]**. Suppose that $x\in\bigcup_{\alpha\in\Delta}A_\alpha$, this implies that there exists an $\alpha_0\in\Delta$ such that $x\in A_{\alpha_0}$ and as A_{α_0} is a $(1,2)S_p$-open set, so there exists a $(1,2)$pre-closed set F in X such that $x\in F\subseteq A_{\alpha_0}\subseteq\bigcup_{\alpha\in\Delta}A_\alpha$; Therefore, $\bigcup_{\alpha\in\Delta}A_\alpha$ is a $(1,2)S_p$-open set.

Remark 2.5. From the above Theorem, we can say that any intersection of $(1,2)S_p$-closed sets of a bitopological space (X,τ_1,τ_2) is $(1,2)S_p$-closed.

The following example shows that the intersection of two $(1,2)S_p$-open sets need not be a $(1,2)S_p$-open set.

Example 2.6. Let $X=\{a,b,c,d\}$ with $\tau_1=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau_2=\{\phi,X,\{a,b,d\}\}$. Then $\{a,c,d\}$ and $\{b,c,d\}$ are $(1,2)S_p$-open sets in X but $\{a,c,d\}\cap\{b,c,d\}=\{c,d\}$ is not $(1,2)S_p$-open set.

Proposition 2.7. A subset G in the bitopological space X is $(1,2)S_p$-open, if and only if for each $x\in G$ there exists a $(1,2)S_p$-open set H such that $x\in H\subseteq G$.
Proof: Let G be a $(1,2)S_p$-open set in X. Then, for each $x\in G$, we have G is a $(1,2)S_p$-open set containing x such that $x\in G\subseteq G$.
Conversely; suppose that for each $x\in G$ there exists a $(1,2)S_p$-open set H such that $x\in H\subseteq G$, then G is the union of $(1,2)S_p$-open sets. Hence by Theorem 2.4, G is $(1,2)S_p$-open.

Remark 2.8. Shareef **[8]** proved that every regular-closed set is S_p-open and every regular-open set is S_p-closed in any topological space. But in bitopological space (X,τ_1,τ_2), we are going to see that the pair $(1,2)$regular-closed sets and $(1,2)S_p$-open sets also the pair $(1,2)$regular-open sets and $(1,2)S_p$-closed sets are independent.

Example 2.9. Let $X=\{a,b,c,d\}$ with $\tau_1=\{\phi,X,\{d\},\{a,d\},\{b,d\},\{a,b,d\}\}$ and $\tau_2=\{\phi,X,\{c\}\}$. Then $(1,2)SO(X)=\{\phi,X,\{d\},\{a,d\},\{b,d\},\{a,b,d\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$, $(1,2)RO(X)=\{\phi,X,\{a,b,d\}\}$, $(1,2)RC(X)=\{\phi,X,\{c\}\}$ and $(1,2)S_pO(X)=\{\phi,X\}$. Then here $\{c\}\in(1,2)RC(X)$ but not a $(1,2)S_p$-open set and $\{a,b,d\}\in(1,2)RO(X)$ but not a $(1,2)S_p$-closed set.

Example 2.10. Let $X=\{a,b,c,d\}$with $\tau_1=\{\phi,X,\{a\},\{a,c\},\{b,c,d\}\}$ and $\tau_2=\{\phi,X,\{b,c\},\{a,b,c\}\}$. Then:
$(1,2)S_pO(X)=\{\phi,X,\{a\},\{a,c\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$,
$(1,2)RO(X)=\{\phi,X,\{a\},\{b,c,d\}\}=(1,2)RC(X)$. Here, $\{a,c\}\in(1,2)S_pO(X)$ but $\{a,c\}\notin(1,2)RC(X)$ and $\{d\}\in(1,2)S_pC(X)$ but $\{d\}\notin(1,2)RO(X)$.

Proposition 2.11. If a space X is ultra T_1-space, then $(1,2)S_pO(X)=(1,2)SO(X)$.

Proof. It is true, since every singleton set is $(1,2)\alpha$-closed in an ultra T_1 space and every $(1,2)\alpha$-closed set is $(1,2)$pre-closed.

In **[7]** it was shown that the family of all $(1,2)$semi-open subsets of a bitopological space (X,τ_1,τ_2) is closed with respect to arbitrary union, but the intersection of any two $(1,2)$semi-open sets need not be (1,2)semi-open. So normally the family of $(1,2)$semi-open sets is not a topology. This gives the following preposition.

Proposition 2.12. If the family of $(1,2)$semi-open sets forms a topology, then the family of all $(1,2)S_p$-open sets is also a topology.
Proof. Obvious.

Corollary 2.13. Let (X,τ_1,τ_2) be a bitopological space and if X be $(1,2)\alpha$-extremally disconnected. Then $(1,2)S_pO(X)$ forms a topology on X.
Proof. Follows from Theorem 1.9 and Proposition 2.12.

Proposition 2.14. If a space X is $(1,2)\alpha$-hyperconnected, then the only $(1,2)S_p$-open sets in X are ϕ and X.
Proof. Suppose that $A \subseteq X$ such that A is $(1,2)S_p$-open set in X. If $A = X$, then there is nothing to prove. If $A \neq X$, then we need to prove $A = \phi$. Since A is $(1,2)S_p$- open set in X, $X\backslash A \subseteq X\backslash F$, but as $X\backslash A$ is $(1,2)$semi-closed, $(1,2)sCl(X\backslash A) = (X\backslash A)$. As X is $(1,2)\alpha$-hyperconnected, then by **[7]**, $(1,2)sCl(X\backslash A) = X = (X\backslash A)$. Thus $X\backslash A = X$, this implies that $A = \phi$. Hence the only $(1,2)S_p$-open sets of X are ϕ and X.

Let us, define a subspace of a bitopological spaces as given below.

Definition 2.15. A bitopological space (Y,σ_1,σ_2) is called a bitopological subspace of a bitopological space (X,τ_1,τ_2), if $Y \subseteq X$ and σ_i is a relative topology to τ_i on Y for $i = 1,2$.

Remark 2.16. Let (Y,σ_1,σ_2) be a bitopological subspace of (X,τ_1,τ_2). Then:

1. If a subset A is $(1,2)S_p$-open set relative to Y, then A need not always be $(1,2)S_p$-open in X.
2. If A is $(1,2)S_p$-open in X, then $A \cap Y$ need not be $(1,2)S_p$-open in Y.

The above results are justified by the following examples:

Example 2.17. Let $X = \{a,b,c,d\}$ and let $\tau_1 = \{\phi, X, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{a,c,d\}, \{a,b,c\},\{a,b,d\}\}$ and $\tau_2 = \{\phi, X\}$. Then $(1,2)S_pO(X) = \{\phi, X, \{b\},\{c\},\{b,c\},\{a,d\},\{a,c,d\},\{a,b,d\}\}$. Let $Y = \{a,b,c\}$, then $\sigma_1 = \{\phi, Y, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $\sigma_2 = \{\phi, Y\}$. Then $(1,2)S_pO(Y) = \{\phi, Y, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$, here $\{a,b\} \in S_pO(Y)$ but not in $(1,2)S_pO(X)$.

Example 2.18. Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi, X, \{c\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$ and $\tau_2 = \{\phi, X\}$. Then $(1,2)S_pO(X) = \{\phi, X, \{c\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$. If $Y = \{a,c,d\}$, then $\sigma_1 = \{\phi, Y, \{a\},\{c\},\{a,c\},\{a,d\}\}$ and $\sigma_2 = \{\phi, Y\}$, now $(1,2)S_pO(Y) = \{\phi, Y, \{c\},\{a,d\}\}$. See $A = \{a,b\} \in (1,2)S_pO(X)$, but $A \cap Y = \{a\} \notin (1,2)S_pO(Y)$.

Definition 2.19. A bitopological space (X,τ_1,τ_2) is said to be τ_1-locally indiscrete if every τ_1-open subset of X is τ_1 -closed.

Proposition 2.20. If a bitopological space X is τ_1-locally indiscrete, then every τ_1-open set is $(1,2)S_p$-open.
Proof: Let A be a τ_1-open set in X. Then $A \subseteq \tau_1\tau_2 - Cl(\tau_1 Int(A))$ this implies that A is $(1,2)$ semi-open set in X. Since X is τ_1-locally indiscrete so by Definition 2.19, A is also τ_1-closed and then it is $\tau_1\tau_2$-closed set too. Thus $A = \tau - Int(A) = \tau_1\tau_2 - Cl(\tau_1 - Int(A))$ this implies that A is $(1,2)S_p$-open set.

3. $(1,2)S_p$-Operations

Definition 3.1. A subset N of a bitopological space (X,τ_1,τ_2) is called $(1,2)S_p$-neighborhood of a subset A of X, if there exists an $(1,2)S_p$-open set U such that $A \subseteq U \subseteq N$.

For any $\{x\} \subseteq X$, we say that N_x represents $(1,2)S_p$-neighborhood of x.

Definition 3.2. A point $x \in X$ is said to be a $(1,2)S_p$-interior point of A, if there exists a $(1,2)S_p$-open set U containing x such that $x \in U \subseteq A$. The union of all $(1,2)S_p$-open sets contained in A is said to be $(1,2)S_p$-interior of A and it is denoted by $(1,2)S_p\text{-int}(A)$.

Proposition 3.3. Let A be any subset of a bitopological space X. If a point x is a $(1,2)S_p$-interior point of A, then there exists a $(1,2)$pre-closed set F of X containing x such that $F \subseteq A$.
Proof. Suppose that $x \in (1,2)S_p\text{-int}(A)$. Then, there exists a $(1,2)S_p$-open set U of X containing x such that $U \subseteq A$. Since U is a $(1,2)S_p$-open set, then there exists a $(1,2)$pre-closed set F containing x such that $F \subseteq U \subseteq A$. This implies that $F \subseteq A$.

Proposition 3.4. If A and B are any subsets of a bitopological space X, then:
(i) $(1,2)S_p\text{-int}(\phi) = \phi$, and $(1,2)S_p\text{-int}(X) = X$.
(ii) $(1,2)S_p\text{-int}(A) \subseteq A$.
(iii) If $A \subseteq B$, then $(1,2)S_p\text{-int}(A) \subseteq (1,2)S_p\text{-int}(B)$.
(iv) $(1,2)S_p\text{-int}(A) \cup (1,2)S_p\text{-int}(B) \subseteq (1,2)S_p\text{-int}(A \cup B)$.
(v) $(1,2)S_p\text{-int}(A \cap B) \subseteq (1,2)S_p\text{-int}(A) \cap (1,2)S_p\text{-int}(B)$.
(vi) $(1,2)S_p\text{-int}(A \backslash B) \subseteq (1,2)S_p\text{-int}(A) \backslash (1,2)S_p\text{-int}(B)$.
Proof. Obvious.

The converse of (iii), (iv), (v) and (vi) need not be true always as shown in the following examples:
Example 3.5: Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi, X, \{a\}, \{c\}, \{b,c,d\}, \{a,c\}\}$ and $\tau_2 = \{\phi, X, \{b,c\}, \{a,b,c\}\}$. Then $(1,2)S_pO(X) = \{\phi, X, \{a\}, \{b,c,d\}\}$.
(iii) Let $A = \{a,b\}$, $B = \{a,c,d\}$. Then $(1,2)S_p\text{-int}(A) = \{a\}$ and $(1,2)S_p\text{-int}(B) = \{a\}$, now $(1,2)S_p\text{-int}(A) = \{a\} = (1,2)S_p\text{-int}(B)$, but $A \nsubseteq B$.
(iv) Let $A = \{b,d\}$ and $B = \{a,c\}$. Then $(1,2)S_p\text{-int}(A) = \phi$ and $(1,2)S_p\text{-int}(B) = \{a\}$, now $(1,2)S_p\text{-int}(A) \cup (1,2)S_p\text{-int}(B) = \{a\}$ and $A \cup B = X$ implies that $(1,2)S_p\text{-int}(A \cup B) = X$, then it is clear that $(1,2)S_p\text{-int}(A \cup B) \nsubseteq (1,2)S_p\text{-int}(A) \cup (1,2)S_p\text{-int}(B)$.
(v) Let $A = \{a,b,d\}$ and $B = \{a,c,d\}$. Then $(1,2)S_p\text{-int}(A) = \{a\}$ and $(1,2)S_p\text{-int}(B) = \{a\}$, now $(1,2)S_p\text{-int}(A) \cap (1,2)S_p\text{-int}(B) = \{a\}$, while $A \cap B = \{c\}$, and then $(1,2)S_p\text{-int}(A \cap B) = \phi$ this implies that $(1,2)S_p\text{-int}(A) \cap (1,2)S_p\text{-int}(B) \nsubseteq (1,2)S_p\text{-int}(A \cap B)$.
(vi) Let $A = \{b,c,d\}$ and $B = \{a, c, d\}$. Then $(1,2)S_p\text{-int}(A) = \{b,c,d\}$ and $(1,2)S_p\text{-int}(B) = \{a\}$, now $(1,2)S_p\text{-int}(A) \backslash (1,2)S_p\text{-int}(B) = \{b,c,d\}$, but $A \backslash B = \{b\}$ and $(1,2)S_p\text{-int}(A \backslash B) = \phi$ this implies that $(1,2)S_p\text{-int}(A) \backslash (1,2)S_p\text{-int}(B) \nsubseteq (1,2)S_p\text{-int}(A \backslash B)$.

Definition 3.6. Let A be a set in a bitopological space X. A point $x \in X$ is in $(1,2)S_p$-closure of A if and only if $A \cap U \neq \phi$, for every $(1,2)S_p$-open set U containing x. The intersection of all $(1,2)S_p$-closed sets F containing A is called the $(1,2)S_p$-closure of A and is denoted by $(1,2)S_p\text{-cl}(A)$.

Proposition 3.7. Let A be any subset of a bitopological space X. If $A \cap F \neq \phi$ for every $(1,2)$pre-closed set F of X containing x, then the point x is in the $(1,2)S_p$-closure of A.
Proof. Let U be any $(1,2)S_p$-open set in X containing x. Then, there exists a $(1,2)$pre-closed set F such that $x \in F \subseteq U$. Now, if by hypothesis A be any subset of X such that $A \cap F \neq \phi$, for every $(1,2)$pre-closed set F of X containing x, then $A \cap U \neq \phi$, for every $(1,2)S_p$-open set U of X containing x; therefore $x \in (1,2)S_p\text{-cl}(A)$.

Theorem 3.8. If F and E are any two subsets of a bitopological space X, then:
(i) $(1,2)S_p\text{-cl}(\phi) = \phi$ and $(1,2)S_p\text{-cl}(X) = X$.
(ii) $F \subseteq (1,2)S_p\text{-cl}(F)$.
(iii) If $F \subseteq E$, then $(1,2)S_p\text{-cl}(F) \subseteq (1,2)S_p\text{-cl}(E)$.

(iv) (1, 2)S_p-cl$(F) \cup$ (1, 2)S_p-cl$(E) \subseteq$ (1, 2)S_p-cl$(F \cup E)$.
(v) (1, 2)S_p-cl$(F \cap E) \subseteq$ (1, 2)S_p-cl$(F) \cap$ (1, 2)S_p-cl(E).
Proof. Obvious.

In general, for any closure operator $cl(F) \cup cl(E) = cl(F \cup E)$ and for most of the closure operators $cl(F) \cap cl(E) \neq cl(F \cap E)$. In the case of (1, 2)$S_p$-closure operator, the equality sign need not hold for both the cases and it is justified by the following example. This obviously leads to the conclusion, that (1,2)S_p-closure is not a Kuratowski's operator.

Example 3.9. In Example 3.5, $(1,2)S_pO(X) = \{\phi, X, \{a,c,d\}, \{b,c,d\}\}$ and $(1,2)S_pC(X) = \{\phi, X, \{a\}, \{b\}\}$. Now if we take $F = \{a\}$ and $E = \{b\}$, then $(1,2)S_p$-cl$(F) = \{a\}$ and $(1,2)S_p$-cl$(E) = \{b\}$, $(1,2)S_p$-cl$(F) \cup (1,2)S_p$-cl$(E) = \{a,b\}$ and $(1,2)S_p$-cl$(F \cup E) = X$. It follows that $(1,2)S_p$-cl$(F) \cup (1,2)S_p$-cl$(E) \neq (1,2)S_p$-cl$(F \cup E)$.
Again if we take $F = \{a,b\}$ and $E = \{a,c,d\}$, we get that $(1,2)S_p$-cl$(F) = X$ and $(1,2)S_p$-cl$(E) = X$, and then $(1,2)S_p$-cl$(F) \cap (1,2)S_p$-cl$(E) = X$, but $(1,2)S_p$-cl$(F \cap E) = \{a\}$. This implies that $(1,2)S_p$-cl$(F) \cap (1,2)S_p$-cl$(E) \neq (1,2)S_p$-cl$(F \cap E)$.

Proposition 3.10. For any subset A of bitopological space X. Then the following statements are true:
(i) $X \backslash$(1, 2)S_p-cl(A) =(1, 2)S_p-int$(X \backslash A)$.
(ii) $X \backslash$(1, 2)S_p-int(A) =(1, 2)S_p-cl$(X \backslash A)$.
(iii) (1, 2)S_p-int$(A) = X \backslash$(1, 2)S_p-cl$(X \backslash A)$.

Definition 3.11. Let A be a subset of a topological space X. A point $x \in X$ is said to be (1, 2)S_p-limit point of A if for each (1, 2)S_p-open set U containing x, $U \cap (A \backslash \{x\}) \neq \phi$. The set of all (1, 2)$S_p$-limit point of A is called (1, 2)S_p-derived set of A and is denoted by (1, 2)S_p-D(A).

Proposition 3.12. Let A be any subset of a bitopological space X. If $F \cap (A \backslash \{x\}) \neq \phi$, for every (1, 2)pre-closed set F containing x, then $x \in$ (1, 2)S_p-D(A).
Proof: Let the hypothesis be satisfied and let U be any (1, 2)S_p-open set containing x. Then there exists a (1, 2) pre-closed set F such that $x \in F \subseteq U$. Now, the hypothesis $F \cap (A \backslash \{x\}) \neq \phi$ implies that $U \cap (A \backslash \{x\}) \neq \phi$, and so $x \in$ (1, 2)S_p-D(A).

Proposition 3.13. If a subset A of a bitopological space X is (1, 2)S_p-closed, then A contains the set of all of it's (1, 2)S_p-limit points.
Proof. Suppose that A is (1, 2)S_p-closed set, then $X \backslash A$ is (1, 2)S_p-open set. Thus A is (1, 2)S_p-closed if and only if each point of $X \backslash A$ has (1, 2)S_p-neighborhood contained in $X \backslash A$, that is if and only if no point of $X \backslash A$ is (1, 2)S_p-limit point of A, or equivalently that A contains each of its (1, 2)S_p-limit points.

Proposition 3.14. Let A and B be subsets of a bitopological space X. If $A \subseteq B$, then (1, 2)S_p-D$(A) \subseteq$ (1, 2)S_p-D(B).
Proof: Obvious.

Theorem 3.15. Let X be a bitopological space and A be a subset of X. Then:
(i) $A \cup$ (1, 2)S_p-D(A) is S_p-closed set.
(ii) (1, 2)S_p-D((1, 2)S_p-D$(A)) \backslash A \subseteq$ (1, 2)S_p-D(A).
(iii) (1, 2)S_p-D$(A \cup$(1,2)S_p-D$(A)) \subseteq A \cup$(1, 2)S_p-D(A).
Proof.
(i) Let $x \notin A \cup$ (1,2)S_p-D(A). Then $x \notin A$ and $x \notin$ (1,2)S_p-D(A). This implies that there exists a (1,2)S_p-open set N_x in X which contains no point of A other than x. But $x \notin A$, so N_x contains no point of A, which implies that $N_x \subseteq X \backslash A$. Again, N_x is a (1,2)S_p-open set and it is a (1, 2)S_p- neighborhood of each of its points. Also, N_x does not contain any point of A implies no point of N_x can be a (1, 2)S_p-limit point of A. Therefore, no point of N_x can belong to (1,2)S_p-D(A), this implies that $N_x \subseteq X \backslash$(1, 2)$S_p$-

D(A). Hence, it follows that $x \in N_x \subseteq (X\backslash A) \cap (X\backslash$(1,2)$S_p$-D$(A)) \subseteq X\backslash(A \cup$(1,2)$S_p$-D$(A))$. Therefore, $A \cup$(1, 2)S_p-D(A) is (1, 2)S_p-closed.

(ii) If $x \in$ (1,2)S_p-D((1,2)S_p-D$(A))\backslash A$ and U is a (1,2)S_p-open set containing x, then $U \cap$((1,2)S_p-D$(A)\backslash\{x\}) \neq \phi$. Let $y \in U \cap$((1,2)S_p-D$(A)\backslash\{x\})$ implies that $y \in U$ and $y \in$(1, 2)S_p-D$(A)\backslash\{x\}$, then $U \cap (A\backslash\{y\}) \neq \phi$. Let $z \in U \cap (A\backslash\{y\})$ and $z \neq x$ for $z \in A$ and $x \notin A$ implies that $U \cap (A\backslash\{x\}) \neq \phi$; therefore, $x \in$(1, 2)S_p-D(A).

(iii) It follows from part (i) and Proposition 3.13.

Theorem 3.16. Let A be a subset of a space X. Then (1, 2)S_p-cl$(A) = A \cup$ (1, 2)S_p-D(A).
Proof. Since (1,2)S_p-D$(A) \subseteq$ (1, 2)S_p-cl(A) and $A \subseteq$ (1, 2)S_p-cl(A) this implies that $A \cup$ (1, 2)S_p-D$(A) \subseteq$ (1, 2)S_p-cl(A). Again, since (1, 2)S_p-cl(A) is the smallest (1, 2)S_p-closed set containing A. But $A \cup$ (1, 2)S_p-D(A) is (1, 2)S_p-closed set containing A. Thus (1, 2)S_p-cl$(A) \subseteq A \cup$ (1, 2)S_p-D(A). Hence (1, 2)S_p-cl$(A) = A \cup$ (1, 2)S_p-D(A).

Theorem 3.17, Let X be any bitopological space and A be a subset of X. Then (1, 2)S_p-int$(A) = A\backslash$(1, 2)S_p-D(A)
Proof: Obvious.

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